Probability Distributions
OR
Theoretical Distributions

① Probability Distribution

Probability distribution of a discrete variable is known as Discrete probability distribution

When a variable is said to be a discrete variable ?

⇒ within a given interval if a variable can assume finite / limited values, It is said to be a Discrete variable

Discrete probability distributions :

① Binomial's Distribution
② Poisson's Distribution
③ Multinomial's Distribution
④ Geometric Distribution
⑤ Rectangular Distribution

(BP MGR)

Probability distribution of a continuous variable is known as continuous prob Distribution

When a variable is said to be a continuous variable ?

⇒ within a given interval if a variable can assume infinite / unlimited values, It is said to be a continuous variable

continuous probability distributions

① Normal Distribution
② Chi-square (χ^2) Distribution
③ t for student's Distribution
④ F – Distribution

(N, Chi, t, F)

② Binomial's Distribution

prob(x) as per Binomial's theorem/approach $= {}^nC_x \, (p)^x \cdot (q)^{n-x}$

where
n = Number of independent & finite trials
x = Number of successes in 'n' trials
$n - x$ = number of failures in 'n' trials
p = probability of success in a single trial
q = probability of failure in a single trial

$p + q = 1.00 = 100\%$

③ 3 coins are tossed, what is probability of getting 2 tails?

classical approach

All possible equally likely outcomes:

HHH, HHT, HTH, HTT
TTT, TTH, THT, THH

outcomes in favour:

TTH, THT, HTT

$$\text{probability} = \left(\frac{\text{No. of outcomes in favour}}{\text{No. of all possible equally likely outcomes}}\right)$$

$= \left(\frac{3}{8}\right) = 0.3750$

$= 37.50\%$

Binomial's approach

$n = 3$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 2$

prob$(x = 2)$

$= {}^nC_x \, p^x \, q^{n-x}$

$= {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{3-2}$

$= {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$

$= 3 \times \frac{1}{4} \times \frac{1}{2}$

$= \frac{3}{8} = 0.3750$

$= 37.50\%$

④ 5 coins are tossed what is probability of getting 3 or 4 or 5 heads?

classical

No of all possible equally likely outcomes $= 32 = 2^5$

outcomes in favour

HHHTT, HHTHT, HHTTH,
HHHHT, HHHTH, HHTHH,
HHHHH, HTHHT, HTHTH,
HTTHH, HTHHH, THHHT,
THTHH, THHTH, THHHH,
TTHHH

probability $= \left(\dfrac{16}{32}\right) = \dfrac{1}{2} = 50\%$

Binomial's

$n = 5,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 3, 4, 5$

$\text{prob}(x \geq 3)$

$= p(x=3) + p(x=4) + p(x=5)$

$= {}^5C_3 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 + {}^5C_4 \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 + {}^5C_5 \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^5 \left({}^5C_3 + {}^5C_4 + {}^5C_5\right)$

$= \frac{1}{32}(10 + 5 + 1) = \frac{16}{32} = \frac{1}{2}$

$= 50\%$

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⑤ 2 dice are rolled. what is probability of getting 5 points on atleast one dice?

classical

No of all possible equally likely outcomes $= 36$

outcomes in favour

(1,5), (2,5), (3,5), (4,5), (5,1)
(5,2), (5,3), (5,4), (5,5), (5,6)
(6,5)

probability $= \dfrac{11}{36} = 30.5555\%$

Binomials

$n = 2,\ p = \frac{1}{6},\ q = \frac{5}{6},\ x = 1, 2$

$\text{prob}(x \geq 1)$

$= p(x=1) + p(x=2)$

$= 1 - p(x=0)$

$= 1 - {}^2C_0 \left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^2$

$= 1 - \left(1 \times 1 \times \frac{25}{36}\right) = 1 - \frac{25}{36}$

$= \frac{11}{36} = 30.5555\%$

6) 3 dice are rolled what is probability of getting 4 points on 2 dice?

Classical

Outcomes infavour:

(4,4,1), (4,4,2), (4,4,3), (4,4,5),
(4,4,6), (4,1,4), (4,2,4), (4,3,4),
(4,5,4), (4,6,4), (1,4,4), (2,4,4),
(3,4,4), (5,4,4), (6,4,4)

No. of all possible equally likely outcomes $= 6^3 = 216$

probability $= \frac{15}{216} = \frac{5}{72}$

$= 6.944444\ \%$

Binomial's

$n = 3,\ p = \frac{1}{6},\ q = \frac{5}{6},\ x = 2$

$P(x = 2)$

$= {}^{3}C_{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1$

$= 3 \times \frac{1}{36} \times \frac{5}{6} = \frac{15}{216} = \frac{5}{72}$

$= 6.9444\ \%$

7) 12 coins are tossed. what is probability of getting:

i) 5 heads: $= {}^{12}C_{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^7 = \frac{792}{4096} = 19.33593751\ \%$

ii) 10 tails: $= {}^{12}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^2 = \frac{66}{4096} = 1.611328125\ \%$

iii) 10 or more heads: $P(x=10) + P(x=11) + P(x=12)$

$= {}^{12}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^2 + {}^{12}C_{11} \left(\frac{1}{2}\right)^{11} \left(\frac{1}{2}\right)^1 + {}^{12}C_{12} \left(\frac{1}{2}\right)^{12} \left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^{12} \left({}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12}\right)$

$= \frac{66 + 12 + 1}{4096} = \frac{79}{4096} = 1.92871\ \%$

iv) at most 1 head

$$= \left(\frac{1}{2}\right)^{12} \left({}^{12}C_0 + {}^{12}C_1\right) = \frac{13}{4096} = 0.31738\ \%$$

v) 5 OR 7 OR 11 heads

$$= P(x=5) + P(x=7) + P(x=11)$$

$$= \left(\frac{1}{2}\right)^{12} \left[{}^{12}C_5 + {}^{12}C_7 + {}^{12}C_{11}\right]$$

$$= \frac{1}{4096}\left(792 + 792 + 12\right) = \left(\frac{1596}{4096}\right) = 38.96484375\ \%$$

vi) atmost 10 heads

$$= \text{prob}\,(x \le 10)$$

$$= 1 - \left[P(x=11) + P(x=12)\right]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^{12}\left({}^{12}C_{11} + {}^{12}C_{12}\right)\right] = 1 - \frac{13}{4096} = \frac{4083}{4096}$$

$$= 99.68262\ \%$$

vii) atleast 4 heads

$$\text{prob}\,(x \ge 4)$$

$$= 1 - \left[P(x=0) + P(x=1) + P(x=2) + P(x=3)\right]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^{12}\left({}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3\right)\right]$$

$$= 1 - \left(\frac{1 + 12 + 66 + 220}{4096}\right) = 1 - \frac{299}{4096} = \frac{3797}{4096}$$

$$= 92.70\ \%$$

viii) <u>all heads</u>

$P(x=12) = {}^{12}C_{12} \left(\frac{1}{2}\right)^{12} \left(\frac{1}{2}\right)^{0} = 1 \times \frac{1}{4096} \times 1$

$= \frac{1}{4096} = 0.0244141\%$

ix) <u>at most 2 tails</u>

$= \text{prob}(x \leq 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= \left(\frac{1}{2}\right)^{12} \left({}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2\right) = \left(\frac{1+12+66}{4096}\right) = \frac{79}{4096}$

$= 1.9287\%$

x) <u>atleast 8 heads</u>

$\text{prob}(x \geq 8)$

$= \left(\frac{1}{2}\right)^{12} \left({}^{12}C_8 + {}^{12}C_9 + {}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12}\right)$

$= \left(\frac{495+220+66+12+1}{4096}\right) = \left(\frac{794}{4096}\right) = 19.3848\%$

(8) 3 dice are rolled, what is probability of getting <u>atleast 3 points</u> on <u>at most 1 dice</u>?

$\Rightarrow$ $n=3,\ p = \frac{4}{6} = \frac{2}{3},\ q = \frac{2}{6} = \frac{1}{3},\ x = 0, 1$

$p(x \leq 1)$

$= P(x=0) + P(x=1)$

$= {}^{3}C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^3 + {}^{3}C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^2$

$= \left(1 \times 1 \times \frac{1}{27}\right) + \left(3 \times \frac{2}{3} \times \frac{1}{9}\right) = \frac{1}{27} + \frac{6}{27} = \left(\frac{7}{27}\right)$

$= 25.9259\%$

(9) 5 dice are rolled what is probability of getting atleast 2 points on atleast 4 dice?

$\Longrightarrow$ $n = 5, \ p = \frac{5}{6}, \ q = \frac{1}{6}, \ x = 4,5$

$$P(x \geq 4) = P(x=4) + P(x=5)$$

$$= {}^5C_4 \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^1 + {}^5C_5 \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)^0$$

$$= \frac{5 \times 625 \times 1}{7776} + \frac{1 \times 3125 \times 1}{7776} = \frac{3125}{7776} + \frac{3125}{7776}$$

$$= \left(\frac{6250}{7776}\right) = 80.3755\%$$

(10) 5 dice are rolled. What is probability of getting at most 4 points on atleast 3 dice?

$\Longrightarrow$ $n = 5, \ p = \frac{4}{6} = \frac{2}{3}, \ q = \frac{1}{3}, \ x = 3,4,5$

$$\text{prob } (x \geq 3)$$

$$= P(x=3) + P(x=4) + P(x=5)$$

$$= {}^5C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2 + {}^5C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^1 + {}^5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^0$$

$$= \frac{10 \times 8 \times 1}{243} + \frac{5 \times 16 \times 1}{243} + \frac{1 \times 32 \times 1}{243}$$

$$= \left(\frac{80 + 80 + 32}{243}\right) = \frac{192}{243} = 79.01234\%$$

(11) 7 dice are rolled what is probability of getting 3 points on 4 dice?

$\Longrightarrow$ $n = 7, \ p = \frac{1}{6}, \ q = \frac{5}{6}, \ x = 4$

$$P(x=4) = {}^7C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3 = \frac{35 \times 1 \times 125}{279936} = \left(\frac{4375}{279936}\right)$$

$$= 1.5629\%$$

(12) 15 dates are selected at random. What is probability of getting 4 Sundays?

$\Rightarrow$ $n = 15$, $p = \frac{1}{7}$, $q = \frac{6}{7}$, $x = 4$

$$prob(x=4) = {}^{15}C_4 \left(\frac{1}{7}\right)^4 \left(\frac{6}{7}\right)^{11}$$

$$= 1365 \times \frac{1}{2401} \times (0.857142857142)^{11}$$

$$= 10.430996634\%$$

(13) 2 dice are rolled. What is probability of getting atleast 5 points on at least one dice?

classical approach | Binomial approach

Classical approach

outcomes in favour

(1,5) (1,6) (2,5) (2,6) (3,5)
(3,6) (4,5) (4,6) (5,1) (5,2)
(5,3) (5,4) (5,5) (5,6) (6,1)
(6,2) (6,3) (6,4) (6,5) (6,6)

$$probability = \frac{20}{36} = \frac{5}{9}$$

$$= 55.5555555\%$$

Binomial approach

$n = 2$, $p = \frac{2}{6} = \frac{1}{3}$, $q = \frac{2}{3}$

$x = 1, 2$

$$prob(x \geq 1)$$

$$= 1 - prob(x = 0)$$

$$= 1 - {}^{2}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^2$$

$$= 1 - \left(1 \times 1 \times \frac{4}{9}\right) = 1 - \frac{4}{9}$$

$$= 5/9 = 55.5555555\%$$

(14) 5 dice are rolled. what is probability of getting atleast 3 points on at most 1 dice?

$\Rightarrow$ $n = 5$, $p = \frac{4}{6} = \frac{2}{3}$, $q = \frac{1}{3}$, $x = 0, 1$

$$prob(x \leq 1) = p(x=0) + p(x=1)$$

$$= {}^{5}C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5 + {}^{5}C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^4 = \frac{1}{243} + \frac{10}{243} = \left(\frac{11}{243}\right)$$

$$= 4.526751\%$$

(15) Overall 70% students passed an exam. Find the probability that in a group of 6 students atleast 5 have passed the exam?

$\Rightarrow$ $n = 6$, $p = 0.70$, $q = 0.30$, $x = 5, 6$

$$Prob(x \geq 5) = P(x=5) + P(x=6)$$

$$= {}^6C_5 (0.70)^5 (0.30)^1 + {}^6C_6 (0.70)^6 (0.30)^0$$

$$= 0.302526 + 0.117649$$

$$= 0.420175 \quad (\text{i.e. } 42.0175\%)$$

(16) An experiment fails once after succeeding 3 times. Find the probability that in 5 experiments we get atleast 4 successes?

$\Rightarrow$ $n = 5$, $p = \frac{3}{4}$, $q = \frac{1}{4}$, $x = 4, 5$

$$P(x \geq 4) = P(x=4) + P(x=5)$$

$$= {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right)^1 + {}^5C_5 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^0$$

$$= \frac{5 \times 81}{1024} + \frac{1 \times 243 \times 1}{1024} = \frac{648}{1024}$$

$$= 63.28125\%$$

(17) 8 dates are selected at random. What is probability of getting 5 Sundays?

$\Rightarrow$ $n = 8$, $p = \frac{1}{7}$, $q = \frac{6}{7}$, $x = 5$

$$Prob(x = 5) = {}^8C_5 \left(\frac{1}{7}\right)^5 \left(\frac{6}{7}\right)^3$$

$$= \frac{56 \times 1 \times 216}{5764801} = \frac{12096}{5764801}$$

$$= 0.2098251\,09\%$$

(18) Find probability that in 13 True - False questions a student gives 11 correct answers?

$\Rightarrow$ $n = 13$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 11$

$$\text{prob}(x = 11) = {}^{13}C_{11} \left(\frac{1}{2}\right)^{11} \left(\frac{1}{2}\right)^{2}$$

$$= 78 \times \frac{1}{2^{13}} = \frac{78}{8192} = 0.952148437\,\%$$

(19) For Binomial's Theorem

i) Trials are Finite.

ii) Trials are Independent

iii) There is some probability of success, failure in each trial.
i.e. $p, q \neq 0$

iv) No. of successes, no. of trials must be a whole number.

v) $x = 0, 1, 2, 3, 4, 5 \text{ ------}, n$

vi) success & failure are mutually exclusive, mutually exhaustive outcomes.

vii) n, p are a main parameters of Binomial's distribution $\therefore$ Binomial's distribution is BI PARAMETRIC distribution

(20) 5 coins are tossed 80,000 times. Find expected frequency of getting 3 heads?

$\Rightarrow$ $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 3$

$$\text{prob}(x = 3) = {}^{5}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{2} = \frac{10}{32} = \frac{5}{16} = 0.3125$$

$$\frac{0.3125}{1} = \frac{?}{80{,}000}$$

$$? = 80{,}000 \times 0.3125 = 25000$$

$$\text{Freq}(x = 3) = N \times \text{prob}(x = 3)$$

$$\text{Freq}(x) = N \times \text{prob}(x)$$

where 'n' trials are repeated 'N' times

(21) 6 coins are tossed 3,20,000 times, Find expected frequency of getting atleast 5 heads?

$\Rightarrow$ $n = 6$, $N = 3,20,000$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 5, 6$

$$\text{Freq}(x \geq 5) = N \times \text{prob}(x \geq 5)$$

$$= 3,20,000 \times \left[\left(\frac{1}{2}\right)^6 \left({}^6C_5 + {}^6C_6\right)\right]$$

$$= 3,20,000 \times \left(\frac{7}{64}\right) = 35,000$$

(22) 7 coins are tossed 12,80,000 times. Find expected frequency of atmost 5 tails?

$\Rightarrow$ $n = 7$, $N = 12,80,000$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 0, 1, 2, 3, 4, 5$

$$\text{Freq}(x \leq 5) = N \times \text{prob}(x \leq 5)$$

$$= N \times \left[1 - \left(P(x=6) + P(x=7)\right)\right]$$

$$= 12,80,000 \left[1 - \left(\frac{1}{2}\right)^7 \left({}^7C_6 + {}^7C_7\right)\right]$$

$$= 12,80,000 \left(1 - \frac{8}{128}\right) = 12,80,000 \times \frac{120}{128}$$

$$= 12,00,000$$

(23) There are 80,000 families with 5 children each. How many families are expected to have 4 boys?

$\Longrightarrow$ $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 4$, $N = 80{,}000$

$$\text{Freq } f(x=4) = N \times \text{prob}(x=4)$$

$$= 80{,}000 \times {}^5C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^1$$

$$= 80000 \times \frac{5}{32} = 12{,}500$$ families are expected to have 4 boys

(24) There are 400,000 families with 4 children each. How many families are expected to have atleast one boy & atleast one girl?

$\Longrightarrow$ $n = 4$, $N = 4{,}00{,}000$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $x = 1, 2, 3$

$$\text{Freq}(1 \le x \le 3) = \left[p(x=1) + p(x=2) + p(x=3)\right] \times N$$

$$= \left[\left(\tfrac{1}{2}\right)^4 \left({}^4C_1 + {}^4C_2 + {}^4C_3\right)\right] \times 400000$$

$$= \left(\frac{4+6+4}{16}\right) \times 400000 = \frac{14}{16} \times 400000$$

$$= 350000 \text{ families}$$

(25) 5 coins are tossed 1280 times. Find expected frequency of 0,1,2,3,4,5 heads. Also find Mean, Variance, SD of distribution.

No. of heads x	prob $(x) = {}^nC_x (p)^x (q)^{n-x}$	Frequency of $x = N \times {}^nC_x (p)^x (q)^{n-x}$
0	${}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$	$1280 \times \frac{1}{32} = 40$
1	${}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 = \frac{5}{32}$	$1280 \times \frac{5}{32} = 200$
2	${}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = \frac{10}{32}$	$1280 \times \frac{10}{32} = 400$
3	${}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{10}{32}$	$1280 \times \frac{10}{32} = 400$
4	${}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 = \frac{5}{32}$	$1280 \times \frac{5}{32} = 200$
5	${}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 = \frac{1}{32}$	$1280 \times \frac{1}{32} = 40$

x	f	$f \cdot x$	x^2	$f \cdot x^2$
0	40	0	0	0
1	200	200	1	200
2	400	800	4	1600
3	400	1200	9	3600
4	200	800	16	3200
5	40	200	25	1000
	1280	3200		9600

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f} = \frac{3200}{1280} = 2.50$$

$$\text{variance} = \left[\frac{\Sigma f x^2}{N} - \left(\frac{\Sigma f x}{N}\right)^2\right]$$

$$= \left(\frac{9600}{1280} - 2.50^2\right)$$

$$= 1.25$$

$$SD = \sqrt{1.25} = 1.118034$$

$$\boxed{\begin{aligned} \text{Mean} &= n \cdot p = 5 \times \tfrac{1}{2} = 2.50 \\ \text{Variance} &= n \cdot p \cdot q = 5 \times \tfrac{1}{2} \times \tfrac{1}{2} = 1.25 \\ SD &= \sqrt{npq} = \sqrt{1.25} = 1.118034 \end{aligned}}$$

(26) **For Binomial's distribution**

n	p	q	Mean $= n \cdot p$	variance $= n \cdot p \cdot q$	SD $= \sqrt{npq}$
80	0.20	0.80	16	12.80	3.57771
100	0.95	0.05	95	4.75	2.17945
500	0.05	0.95	25	23.75	4.8734
100	0.80	0.20	80	16	4.00
250	0.12	0.88	30	26.40	5.1381
60	0.3333333	0.6666666	20	13.33333	3.6515
30	0.05	0.95	1.50	1.425	1.1937
200	0.1296	0.8704	25.92	22.560768	4.7498
300	0.6638	0.3362	199.14	66.950868	8.1824
500	0.8077	0.1923	403.85	77.660355	8.8125
20	0.8122	0.1878	16.244	3.0506232	1.746603
2000	0.80	0.20	1600	320	17.8885
60	0.08	0.92	4.80	4.416	2.101428
500	0.01	0.99	5	4.95	2.22486
1000	0.50	0.50	500	250	15.8114
2000	0.80	0.20	1600	320	17.8885

Mean is always greater than variance

i.e. $(n \cdot p) > (n \cdot p \cdot q)$

(27) For Binomial's Distribution If $n = 40$, $p = 0.01$

Find Mean, SD, variance

$\Rightarrow$

Mean $= n \cdot p = 40 \times 0.01 = 0.40$

Variance $= n \cdot p \cdot q = 40 \times 0.01 \times 0.99 = 0.3960$

SD $= \sqrt{npq} = \sqrt{0.3960} = 0.6292853089$

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	Binomial's distribution is said to be
If $p = q = 0.50$	$\Rightarrow$ Symmetrical
If $p \neq q$	$\Rightarrow$ Asymmetrical OR Non-symmetrical

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For Binomial's distribution

p	q	variance
0.20	0.80	0.16n
0.10	0.90	0.09n
0.30	0.70	0.21n
0.95	0.05	0.0475n
0.50	0.50	0.25n
0.85	0.15	0.1275n
0.99	0.01	0.0099n

Variance attains its maximum value when $p = q = 0.50$ (i.e. For symmetrical distribution)

Max possible variance of Binomial's distribution $= 0.25n = \left(\frac{n}{4}\right)$

(30) For Binomial's distribution If $n = 20$ then Find Mean, variance, SD if distn is symmetrical.

$\Rightarrow$ As distribution is symmetrical, $p = q = 0.50$

$$\text{Mean} = n \cdot p = 20 \times 0.50 = 10$$

$$\text{Variance} = n \cdot p \cdot q = 20 \times 0.50 \times 0.50 = 5$$

$$SD = \sqrt{npq} = \sqrt{5} = 2.23607$$

(31) Find Mean, SD, variance of Binomial's distribution If $n = 25$, $p = 0.30$

$\Rightarrow$

$$\text{Mean} = n \cdot p = 25 \times 0.30 = 7.50$$

$$\text{variance} = n \cdot p \cdot q = 25 \times 0.30 \times 0.70 = 5.25$$

$$SD = \sqrt{npq} = \sqrt{5.25} = 2.29129$$

(32) If mean = 30, variance = 20 Find n, p, q, SD for Binomial's distribution

$\Rightarrow$

$$n \cdot p = 30$$

$$n \cdot p \cdot q = 20$$

$$30 \times q = 20$$

$$\boxed{q = 0.6666666666 = 2/3}$$

$$\therefore \boxed{p = 0.3333333333 = 1/3}$$

$$n \cdot p = 30$$

$$n \times 0.3333333333 = 30$$

$$\boxed{n = 90}$$

$$SD = \sqrt{\text{variance}} = \sqrt{20} = 4.472136$$

(33) If $n = 120$ Find max possible variance of Binomial's distribution.

$\Rightarrow$ Binomial's distribution attains max. value of variance when $p = q = 0.50$

$\therefore$ Max. possible variance $= \frac{n}{4} = \frac{120}{4} = 30.00$

(34) Binomial distribution has attained max possible of variance (i.e. $0.25n$) then we can say that distribution is

(a) ~~symmetrical~~ (b) asymmetrical (c) Both (d) None

(35) 5 dice are rolled. What is probability of getting atmost 5 points on at most 3 dice?

$\Longrightarrow$ $n = 5$, $p = \frac{5}{6}$, $q = \frac{1}{6}$, $x = 0, 1, 2, 3$

$$\text{prob}(x \le 3) = 1 - [P(x=4) + P(x=5)]$$

$$= 1 - \left[{}^{5}C_{4} \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^1 + {}^{5}C_{5} \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)^0 \right]$$

$$= 1 - \left(\frac{5 \times 625}{7776} + \frac{1 \times 3125}{7776} \right)$$

$$= 1 - \frac{6250}{7776} = \frac{1526}{7776} = 19.6245\%$$

(36) In a village 20% population suffer from a disease. Find the probability that in a group of 10 people 8 or more suffer from that disease?

$\Longrightarrow$ $n = 10$, $p = 0.20$, $q = 0.80$, $x = 8, 9, 10$

$$P(x \ge 8)$$

$$= P(x=8) + P(x=9) + P(x=10)$$

$$= {}^{10}C_{8} (0.20)^8 (0.80)^2 + {}^{10}C_{9} (0.20)^9 (0.80)^1 + {}^{10}C_{10} (0.20)^{10} (0.80)^0$$

$$= 0.000073728 + 0.000004096 + 0.0000001024$$

$$= 0.0000779264\%$$

(37) There are 800000 families with 6 children each. How many families are expected to have 5 or more Girls ?

$\Longrightarrow$ $n = 6,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 5, 6,\ N = 800000$

$$\text{Freq}(x \geq 5) = N \times \text{prob}(x \geq 5)$$

$$= 800000 \left[\left(\frac{1}{2}\right)^6 \left({}^6C_5 + {}^6C_6 \right) \right]$$

$$= 800000 \times \frac{7}{64} = 87500 \text{ families}$$

(38) 15 dates are selected at random, what is probability of getting 5 Tuesdays ?

$\Longrightarrow$ $n = 15,\ p = \frac{1}{7},\ q = \frac{6}{7},\ x = 5$

$$\text{Prob}(x = 5) = {}^{15}C_5 \left(\frac{1}{7}\right)^5 \left(\frac{6}{7}\right)^{10}$$

$$= \left(3003 \times \frac{1}{16807} \times \frac{60466176}{282475249} \right)$$

$$= 3.8246\ \%$$

(39) For Binomial's distribution

x = Discrete variable

$x = 0, 1, 2, 3, 4 \ldots\ldots\ldots, n$

Max value of $x = n$

(40) How to find 'mode' of Binomial's distribution?

First, Find the value of $(n+1)p$

Value of $(n+1)p$ is an integer

↓

Distri. is Bi-modal

↓

modes are $(n+1)p$ and $[(n+1)p - 1]$

Value of $(n+1)p$ is Non-integer

↓

Distri. is uni-modal.

↓

Mode = Largest Integer contained in value of $(n+1)p$

(41) If $n = 16$, $p = \frac{1}{3}$ Find mode for Binomial's distribution

$\Longrightarrow$ $(n+1) \times p = (16+1) \times \frac{1}{3} = 5.666666$

$\therefore$ Distri. is uni-modal.

Mode = 5.00

(42) If $n = 19$, $p = 0.20$. Find mode for Binomial's distribution

$\Longrightarrow$ $(n+1) \times p = (19+1) \times 0.20 = 4$

Distribution is Bi-modal

Modes are : 4, 3

43

Summary of Binomial's distribution

1. It is derived from Bernoulli's trials
2. $prob(x) = {}^nC_x \, (p)^x (q)^{n-x}$
3. $p+q = 1.00$, $p = 1-q$, $q = 1-p$
4. $Freq(x) = N \times prob(x)$
   $= N \times {}^nC_x \, (p)^x (q)^{n-x}$
   where 'n' trials are repeated 'N' times
5. $x = 0, 1, 2, 3, 4, \ldots, n$
   max. value of $x = n$
6. It is a Discrete probability distribution.
   (Distri. applicable only for Discrete variable)
7. Mean $= n \cdot p$, variance $= n \cdot p \cdot q$, SD $= \sqrt{npq}$
8. If $(n+1)p$ is integer : Distri. is Bimodal
   & Modes are $(n+1)p$, $[(n+1)p - 1]$
9. If $(n+1)p$ is Non integer : Distri. is uni-modal
   Mode = Largest integer in value of $(n+1)p$
10. when $p = q = 0.50 \implies$ Distri. is symmetrical
    when $p \neq q \implies$ Distri. is asymmetrical or Non-symmetrical
11. Mean is always greater than variance
    as $(np) > (npq)$
12. Max possible variance $= 0.25n = (n/4)$
    & variance becomes maximum when $p = q = 0.50$
13. Trials are independent, finite
14. There is some probability of success, failure in each trial. i.e. $p, q \neq 0$
15. success & failure are mutually exclusive & mutually exhaustive outcomes
16. n, p are 2 main parameters of Binomial's distribution $\therefore$ It is Bi-parametric distribution.

44) 1.50% of the bulbs manufactured by X R Ltd are known to be defective. Find probability that in a sample of 200 bulbs there are 2 defective bulbs?

Binomial's approach | Poisson's Model

Binomial's approach

$n = 200$, $p = 0.015$, $q = 0.985$

$x = 2$

$prob(x = 2)$

$= {}^{200}C_2 \, (0.015)^2 \, (0.985)^{198}$

$= 19900 \times 0.000225 \times 0.05010674351$

$= 22.441\%$

Poisson's Model

$m = \text{mean} = 200 \times 0.015$

$= 3$

$$prob(x) = \left(\frac{e^{-m} \times m^x}{x!} \right)$$

where

m = mean of Distribution

$x = 0, 1, 2, 3, 4, \text{------}, \infty$

e = exponential factor

$= 2.7183$ (approx)

$prob(x = 2)$

$$= \frac{e^{-3} \times 3^2}{2!} = \frac{4.50}{2.71833^3}$$

$= 22.4037\%$

45) $prob(x)$ as per poisson's model

$$= \left(\frac{e^{-m} \times m^x}{x!} \right)$$

Where

e = exponential factor

$= 2.7183$ (approx)

$x = 0, 1, 2, 3, \text{----}, \infty$

m = mean of the distribution

(46) If mean of poisson's Distribution is 5. Find prob $(x=3)$, prob $(x \le 1)$

$\Rightarrow$

① $\text{prob}(x=3) = \dfrac{e^{-5} \times 5^3}{3!} = \left(\dfrac{20.8333333}{2.71828^5}\right)$

$= 14.037\%$

② $\text{prob}(x \le 1) = P(x=0) + P(x=1)$

$= \dfrac{e^{-5} \times 5^0}{0!} + \dfrac{e^{-5} \times 5^1}{1!}$

$= e^{-5}\left(\dfrac{5^0}{0!} + \dfrac{5^1}{1!}\right) = \dfrac{6}{2.71828^5}$

$= 4.0426\%$

(47) 'm' is only one parameter of poisson's distribution

∴ poisson's distribution is said to be Uni-parametric distribution.

	Discrete/continuous	Parameters	Type
Binomial's	Discrete	n, p	BI PARAMETRIC
poisson's	Discrete	m (Mean)	UNI PARAMETRIC
Normal	continuous	μ, σ^2 (Mean, variance)	BI PARAMETRIC

48) For a poisson's variate If $m = 6$

Find prob $(x = 4)$, prob $(x = 5)$

$\Rightarrow$

$$P(x=4) = \left(\frac{e^{-6} \times 6^4}{4!}\right) = \frac{54}{2.7183^6}$$

$$= 13.3847\ \%$$

$$P(x=5) = \left(\frac{e^{-6} \times 6^5}{5!}\right) = \frac{64.80}{2.7183^6}$$

$$= 16.06166982\ \%$$

49) For poisson's variate If $m = 4$

Find prob $(-2.86 < x \leq 1)$

$\Rightarrow$

$$P(x=0) + P(x=1)$$

$$= \frac{e^{-4} \times 4^0}{0!} + \frac{e^{-4} \times 4^1}{1!}$$

$$= e^{-4}(1+4) = \left(\frac{5}{2.7183^4}\right) = 9.157575\ \%$$

50) For poisson's variate If $m = 3$

Find prob $(-12.86 < x < 3)$

$\Rightarrow$

$$P(x=0) + P(x=1) + P(x=2)$$

$$= e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right)$$

$$= \frac{1}{2.7183^3}(1 + 3 + 4.50) = \left(\frac{8.50}{2.7183^3}\right)$$

$$= 42.32\ \%$$

(51) If $m = 4$ for poisson's variate, Find prob $(x \geq 1)$, prob $(x > 1)$

$\Longrightarrow$ (1) prob $(x \geq 1) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + \cdots\cdots$

$$= 1 - P(x=0)$$

$$= 1 - \left(\frac{e^{-4} \times 4^0}{0!}\right)$$

$$= 1 - \left(\frac{1}{2.7183^4}\right) = 98.1685\%$$

(2) prob $(x > 1) = P(x=2) + P(x=3) + P(x=4) + \cdots\cdots$

$$= 1 - \left[P(x=0) + P(x=1)\right]$$

$$= 1 - \left[e^{-4}\left(\frac{4^0}{0!} + \frac{4^1}{1!}\right)\right]$$

$$= 1 - \left(\frac{5}{2.7183^4}\right) = 90.8424\%$$

(52) If $m = 6$ for poisson's variate
Find prob $(2.30 < x < 4)$

$\Longrightarrow$ $P(x=3)$

$$= \left(\frac{e^{-m} \times m^x}{x!}\right) = \frac{e^{-6} \times 6^3}{3!}$$

$$= \frac{36}{2.7183^6} = 8.9235\%$$

(53) For a poisson's variate If mean = m = 4

Find (1) prob $(-23.86 < x < 1)$ (4) prob $(x \geq 1)$

(2) prob $(2 \leq x \leq 3)$

(3) prob $(-1.86 < x \leq 1)$

$\Longrightarrow$

(1) $P(-23.86 < x < 1) = P(x = 0)$

$$= \frac{e^{-4} \times 4^0}{0!} = \frac{1}{e^4}$$

$$= \frac{1}{2.71834^4} = 0.018315$$

$$= 1.8315\%$$

(2) $P(2 \leq x \leq 3) = P(x=2) + P(x=3)$

$$= \frac{e^{-4} \times 4^2}{2!} + \frac{e^{-4} \times 4^3}{3!}$$

$$= \frac{1}{e^4}\left(\frac{16}{2} + \frac{64}{6}\right) = \frac{18.66666}{2.71839}$$

$$= 34.1883\%$$

(3) $P(-1.86 < x \leq 1) = P(x=0) + P(x=1)$

$$= \frac{e^{-4} \times 4^0}{0!} + \frac{e^{-4} \times 4^1}{1!}$$

$$= \frac{1}{2.7183^4}(1 + 4)$$

$$= 9.1576\%$$

(4) $P(x \geq 1) = P(x=1) + P(x=2) + P(x=3) + \cdots\cdots$

$$= 1 - P(x = 0)$$

$$= 1 - 0.018315 = 0.981685$$

$$= 98.1685\%$$

(54) For a poisson's distribution
prob $(x = 3)$ = prob $(x = 4)$
Find value of 'm'

$\Rightarrow$ $P(x=3) = P(x=4)$

$$\frac{\cancel{e^{-m}} \times m^3}{3!} = \frac{\cancel{e^{-m}} \times m^4}{4!}$$

$$\frac{\cancel{m^3}}{6} = \frac{\cancel{m^3} \times m}{24}$$

$$\frac{24}{6} = m$$

$\therefore \; m = 4$

(55) For a poisson's distribution
prob $(x = 4)$ = prob $(x = 5)$
Find value of 'm'

$\Rightarrow$ prob $(x=4)$ = prob $(x=5)$

$$\frac{\cancel{e^{-m}} \times \cancel{m^4}}{4!} = \frac{\cancel{e^{-m}} \times \cancel{m^5}^{\,m}}{5!}$$

$$\frac{1}{24} = \frac{m}{120}$$

$\therefore \; m = \frac{120}{24} = 5$

(56) For a poisson's variate If $m = 5$
Find prob $(-3.86 < x \leq 3)$

$\Rightarrow$ $P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!}\right) = \left(\frac{1 + 5 + 12.50 + 20.8333}{2.71835}\right)$$

$$= \left(\frac{39.33333}{2.71835}\right) = 26.50171$$

(57) For poisson's Distribution

① $x = 0, 1, 2, 3, \ldots\ldots, \infty$

x is a discrete variable as within a given interval it can take/assume finite values

② Mean = m

③ variance = m

④ SD = $\sqrt{m}$

⑤ Mean = variance = m

⑥ This is preferred over Binomial's distribution when there are Large no. of trials or probability of success/failure is very small.

(58) SD of a poisson's variate is 1.7320508. Find prob $(x = 5)$

$\Rightarrow$ $\sqrt{m} = 1.7320508$

$m = 3$

$$\text{prob}(x = 5) = \left(\frac{e^{-3} \times 3^5}{5!}\right) = \left(\frac{2.025}{2.7183^3}\right) = 10.0816791\%$$

(59) 1.75% of Airline suffer from Engine failure. Find the probability that in 400 flights there will be 3 Engine failures?

$\Rightarrow$

Binomial's model

$n = 400$, $p = 0.0175$

$q = 0.9825$, $x = 3$

$\text{prob}(x = 3)$

$= {}^{400}C_3 (0.0175)^3 (0.9825)^{397}$

$= 10586800 \times 0.000005359375$

$\times 0.000898289812$

$= 5.10\%$

Poisson's model

$m = 400 \times 0.0175 = 7$

$$P(x = 3) = \frac{e^{-7} \times 7^3}{3!}$$

$$= \left(\frac{57.16666666}{2.7183^7}\right) = 5.21\%$$

Mode of Poisson's Distribution

Find value of 'm'

- 'm' is non-integer
  - Distribution is uni-modal
  - Mode = Largest integer contained in value of 'm'
- 'm' is an integer
  - Distribution is Bi-modal
  - Modes are m & $(m-1)$

61 Difference between Binomial's Distribution & Poisson's Distribution.

⟶ Both are Discrete probability distributions

Point of Difference	Binomial's Distribution	Poisson's Distribution
Formula of prob(x)	$prob(x) = {}^{n}C_{x}\,(p)^{x}\cdot(q)^{n-x}$	$prob(x) = \left(\frac{e^{-m}\times m^{x}}{x!}\right)$
Value of x	$x = 0, 1, 2, 3, \ldots\ldots n$	$x = 0, 1, 2, 3, \ldots\ldots \infty$
parameters	n, p	m
Type	Bi-parametric	uni-parametric
Mean	$n \cdot p$	m
variance	$n \cdot p \cdot q$	m
standard Deviation	$\sqrt{n \cdot p \cdot q}$	$\sqrt{m}$
Relation betn Mean & variance	Mean $>$ variance as $np > npq$	Mean $=$ variance $= m$
Mode is based on value	$(n+1)p$	m

62. Probability of a bomb hitting the Bridge is 20%. 5 bombs are thrown, 3 bombs are sufficient to destroy the Bridge. Find probability that Bridge will be destroyed?

$\Longrightarrow$ $n = 5$, $p = 0.20$, $q = 0.80$, $x = 3, 4, 5$

$$P(x=3) + P(x=4) + P(x=5)$$

$$= {}^5C_3 (0.20)^3 (0.80)^2 + {}^5C_4 (0.20)^4 (0.80)^1 + {}^5C_5 (0.20)^5 (0.80)^0$$

$$= 0.0512 + 0.0064 + 0.00032$$

$$= 5.792\%$$

63. For Binomial's Distribution if $n = 6$ and $4 \times prob(x=4) = prob(x=2)$. Find values of p, q, Mean, Mode, variance, SD.

$\Longrightarrow$

$$4\, prob(x=4) = prob(x=2)$$

$$4 \times {}^6C_4\, p^4 q^2 = {}^6C_2\, p^2 q^4$$

$$4 \times \cancel{15} \times p \times p \times \cancel{p^2} \times \cancel{q^2} = \cancel{15} \times \cancel{p^2} \times \cancel{q^2} \times q^2$$

$$4p^2 = q^2$$

$$(2p)^2 = q^2$$

$$2p = q$$

$$2p = 1 - p$$

$$3p = 1$$

$$\therefore p = \frac{1}{3},\ q = \frac{2}{3}$$

Mean $= n \cdot p = 6 \times \frac{1}{3} = 2$

variance $= n \cdot p \cdot q$

$= 6 \times \frac{1}{3} \times \frac{2}{3}$

$= 1.333333$

SD $= \sqrt{1.333333}$

$= 1.1547$

Mode $=$ Largest integer in $(6+1) \times \frac{1}{3}$

$= 2.00$

64. For Binomial's Distribution $n = 24$, $p = 0.20$

Find

Mean $= n \cdot p = 24 \times 0.20 = 4.80$

Variance $= n \cdot p \cdot q = 24 \times 0.20 \times 0.80 = 3.84$

SD $= \sqrt{npq} = \sqrt{3.84} = 1.9596$

Modes $=$ 5 and 4

$(n+1)p = (24+1) \times 0.20 = 5$

65. Which of the following distribution is Bi-parametric

(a) Binomial's (b) Normal (c) Both of these (a & b)

(d) Poisson's

66. Which of the following distribution is uni-parametric

(a) Binomial's (b) Normal (c) Both of these (a & b)

(d) poisson's

67. parameters of Binomial's Distribution are :

(a) n, p (b) p, q (c) n, x (d) p, x

68. The probability distribution may be ______

(a) Discrete (b) continuous (c) (a) or (b) (d) (a) and (b)

(69) The theoretical distribution ______

(a) does not exist (b) exists in real life

~~(c)~~ exists only in theory (d) none of these

(70) The parameters of normal distribution are

~~(a)~~ μ, σ^2 (b) n, p

(c) m, n (d) None of these

(71) The most important continuous probability distribution is ______ Distribution.

(a) t (b) F (c) χ^2 ~~(d)~~ Normal

(72) Match the following

Mean	$\sqrt{npq}$
SD	$(n \times p)$
variance	(n, p)
parameters	(npq)

(73) For Binomial's distribution mean = 9 variance = 2.25

Find n, p, q

$\Longrightarrow$ $n \cdot p = 9$ $\quad$ $n \cdot p \cdot q = 2.25$

$9 \times q = 2.25$

$\therefore q = 0.25$ $\quad \therefore p = 0.75$ $\quad \therefore n = 12$

74) 13 coins are tossed 1,22,880 times. Find expected freq. of atleast 12 tails?

$\Longrightarrow$

$$\text{Freq}(x \geq 12) = N \times \text{prob}(x \geq 12)$$

$$= 1{,}22{,}880 \times \left[P(x=12) + P(x=13) \right]$$

$$= 1{,}22{,}880 \times \left[{}^{13}C_{12} \left(\tfrac{1}{2}\right)^{12} \left(\tfrac{1}{2}\right)^{1} + {}^{13}C_{13} \left(\tfrac{1}{2}\right)^{13} \left(\tfrac{1}{2}\right)^{0} \right]$$

$$= 1{,}22{,}880 \times \left(\frac{13}{8192} + \frac{1}{8192} \right)$$

$$= 1{,}22{,}880 \times \frac{14}{8192} = 210$$

75) If $n = 51$, $p = 0.25$ then Binomial's distribution is :

(a) uni-modal ~~(b)~~ Bi-modal

(c) Tri-modal (d) can't say

$(n+1)p = 13$ $\therefore$ modes are 13, 12

76

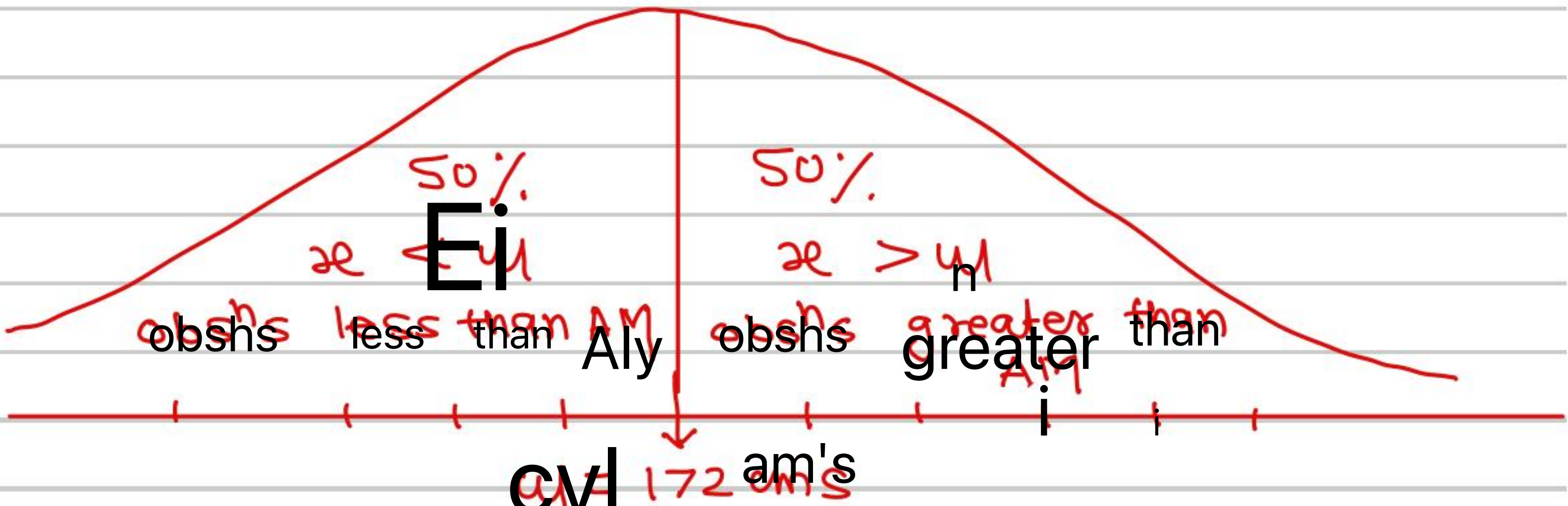
NORMAL DISTRIBUTION

i) It is a probability distribution applicable to the continuous variables like Age, Height, Income, weight etc.

ii) It is derived by 'Karl Gauss'

∴ It is known as Gaussian's Theorem

iii) Normal distribution is based on 'Assumption of Normality'

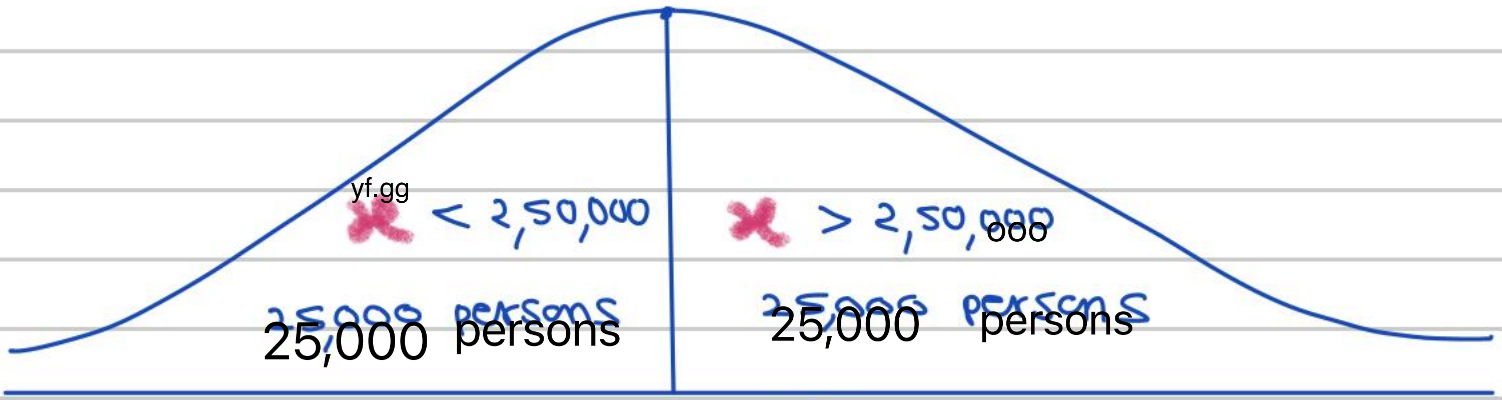


Normal curve is a Bell-shaped curve symmetrical about AM

iv) a continuous variable is said to be normally distributed if 50% of the observations are more than AM and 50% of the observations are less than AM.

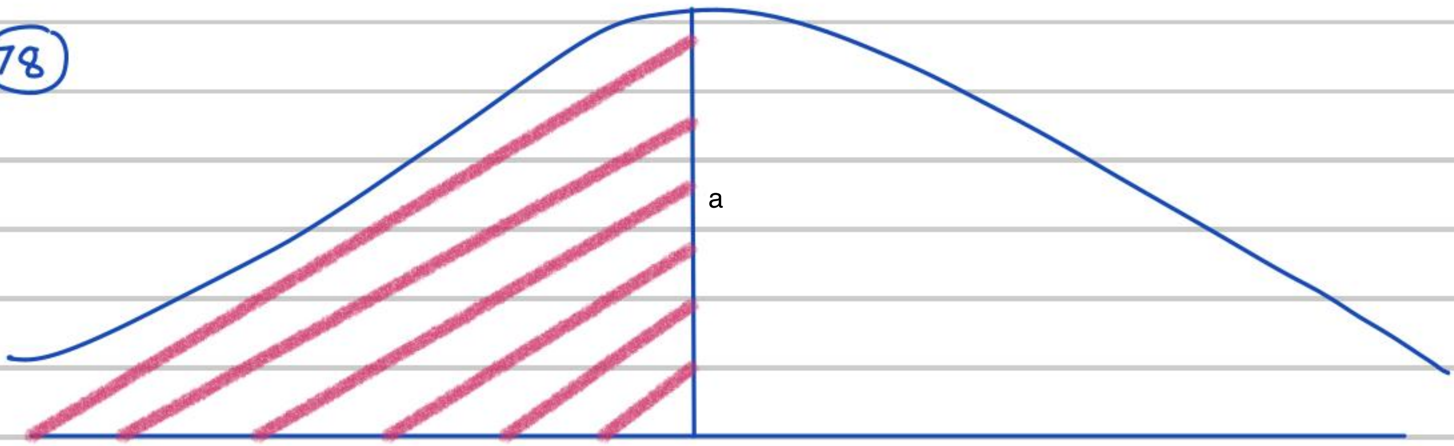
(77) If there are 50,000 people in a city and Avg income per person is ₹ 2,50,000/- then

Income of 50,000 people is said to be normally distributed. If 25,000 people earn more than ₹ 2,50,000 and 25,000 people earn less than ₹ 2,50,000.



$x = \mu = 2,50,000$

(78)



Total area under normal curve $= 1.00 = 100\%$

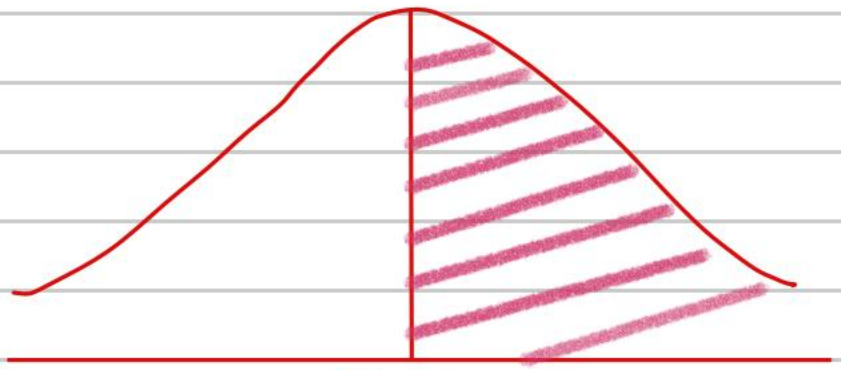
In above diagram

shaded area $= 0.50 = \text{prob}(x \leq \mu)$

unshaded area $= 0.50 = \text{prob}(x > \mu)$

(79) Wages of workers in a factory are normally distributed with AM of ₹ 8,000 and SD of ₹ 750. If one worker is randomly selected, find the probability that he earns

(a) more than ₹ 8000



$x = \mu$

$x = 8{,}000$

prob $(x > 8000)$

= Shaded area

= 0.50 = 50%.

(b) Less than ₹ 8000

$x = \mu$

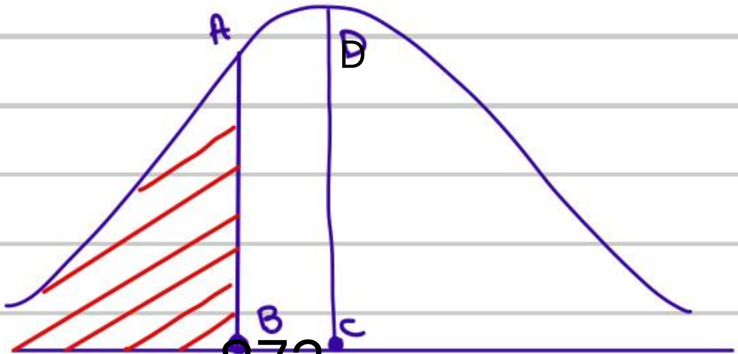
$x = 8{,}000$

prob $(x < 8000)$

= Shaded area

= 0.50 = 50%.

(c) Less than ₹ 7600



$x = 7600$ $x = 8000$

$z = \frac{7600 - 8000}{750}$ $z = \frac{8000 - 8000}{750} = 0$

$= -0.53$

prob $(x < 7600)$

= Shaded area

= 0.50 − A(ABCD)

= 0.50 − 0.20194

= 0.29806 i.e. 29.806%.

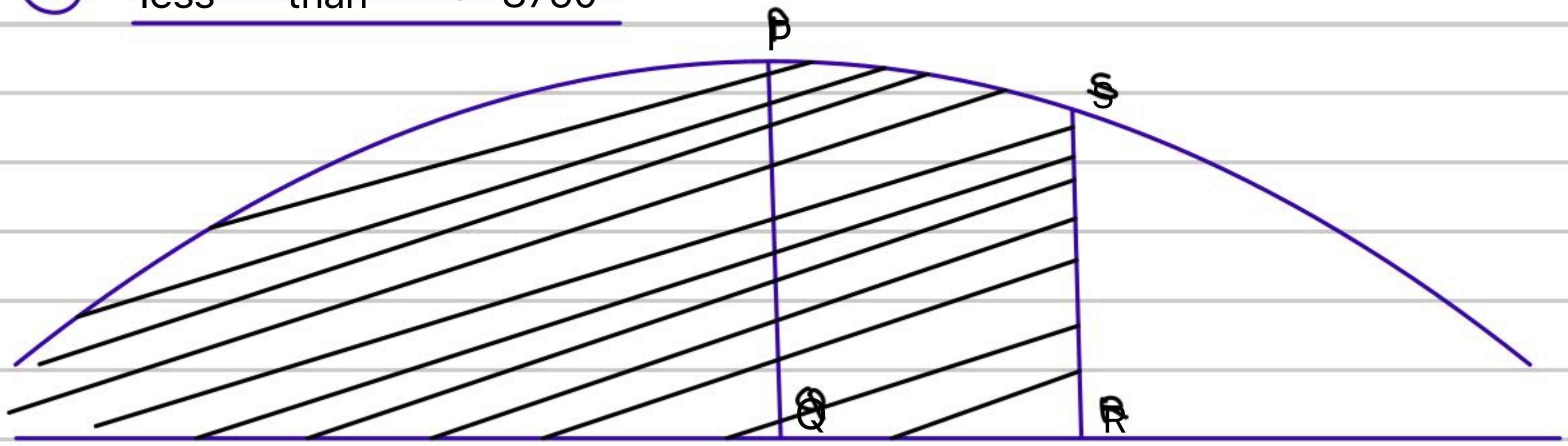
29.806% of Total workers earn less than ₹ 7600/-

z = normal curve coefficient $= \left(\dfrac{x - \mu}{\sigma}\right)$

μ = AM of Distribution = 8,000

σ = SD of Distribution = 750

(d) less than ₹ 8750



$x = \mu = 8000$ $\qquad$ $x = 8750$

$z = 0$ $\qquad$ $z = \dfrac{8750 - 8000}{750}$

$z = 1.00$

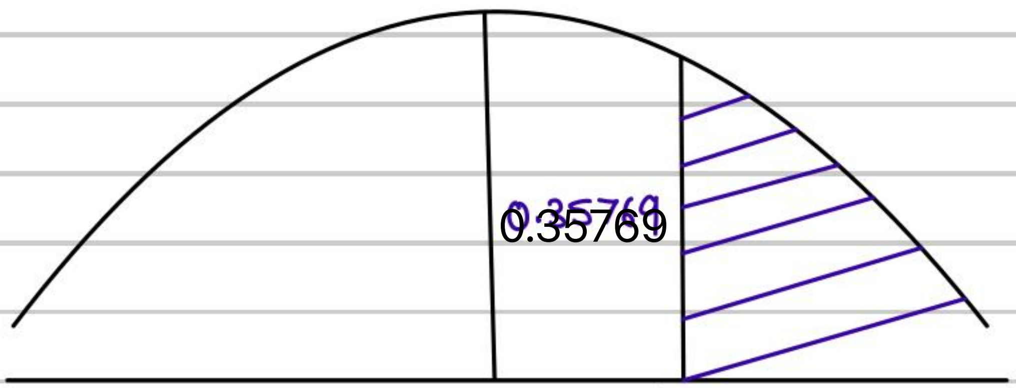
prob $(x \le 8750)$ = A(PQRS) + 0.50

= 0.34134 + 0.50 = 0.84134

= 84.134%

84.134% of Total workers earn less than ₹8750/-

(e) more than ₹ 8800/-



prob $(x > 8800)$

= 0.50 − 0.35769

= 0.14231

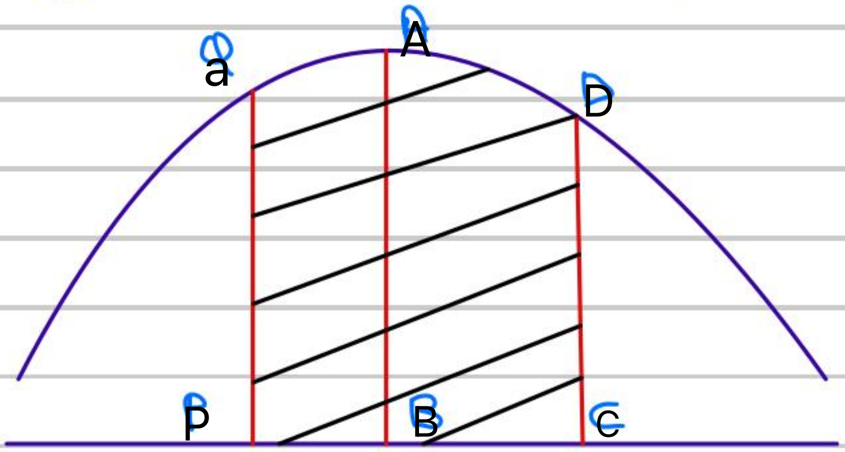
= 14.231%

$x = 8000$ $\qquad$ $x = 8800$

$z = 0$ $\qquad$ $z = \dfrac{8800 - 8000}{750}$

$z = 1.07$

(f) between ₹ 7800 & ₹ 8400



$x = 7800$ $\quad x = 8000$ $\quad x = 8400$

$z = \frac{7800 - 8000}{750}$ $\quad z = 0$ $\quad z = \frac{8400 - 8000}{750}$

$z = -0.27$ $\quad z = 0.53$

prob $(7800 < x < 8400)$

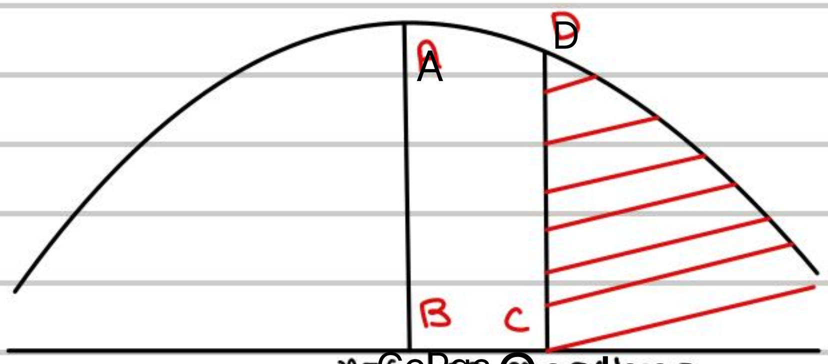
$= A(QBPQ) + A(ABCD)$

$= 0.10642 + 0.20194$

$= 0.30836 = 30.836\%$

(80) Weight of 10000 students is normally distributed with AM, SD of 60 kgs, 8 kgs resp. Find probability that a student randomly selected has weight of more than 64 kgs?

$\Longrightarrow$



$x = 60$ kgs $\quad x = 64$ kgs

$z = 0$ $\quad z = \frac{64 - 60}{8}$

$z = 0.50$

prob $(x > 64)$

$= 0.50 - A(ABCD)$

$= 0.50 - 0.19146$

$= 0.30854$ (i.e. 30.854%)

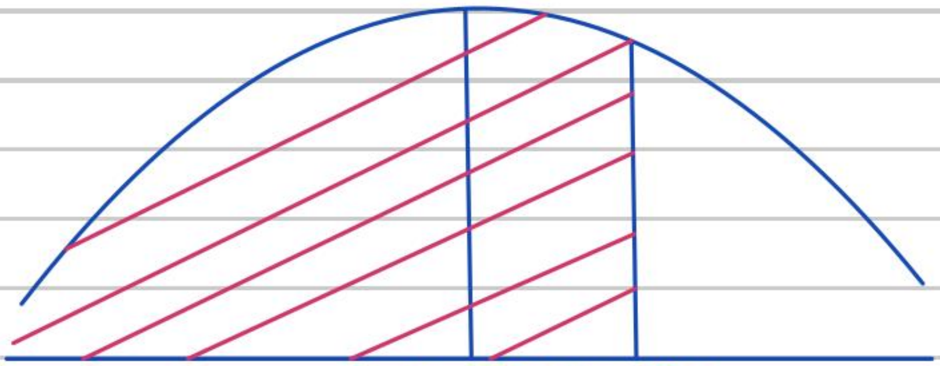
30.854% of Total students has weight of more than 64 kgs.

Approx weight of 3085 students is more than 64 kgs

(81) If $\mu = 12{,}000$, $\sigma = 950$

Find prob $(x < 13000)$, prob for $(x > 13000)$

$\Longrightarrow$



$x = 12000$ $\quad$ $x = 13000$

$z = 0$ $\quad$ $z = \dfrac{13000 - 12000}{950}$

$z = 1.05$

prob $(x < 13000)$

$= 0.50 + 0.35314$

$= 0.85314$

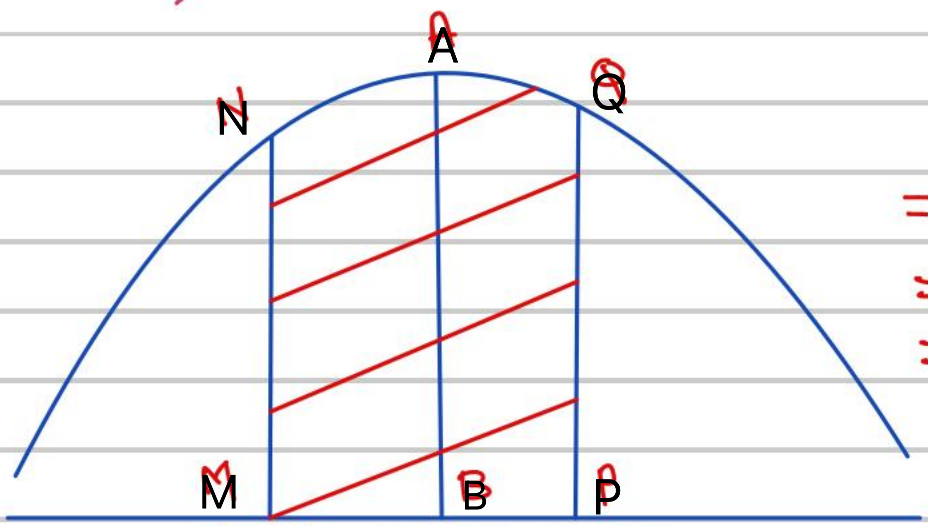
i.e. 85.314%

$\therefore$ prob $(x > 13000)$

$= 14.686\%$

(82) If $\mu = 85$, $\sigma = 13.20$ Find prob $(80 < x < 87)$

$\Longrightarrow$



$x = 80$ $\quad$ $x = 85$ $\quad$ $x = 87$

$z = \dfrac{80 - 85}{13.20}$ $\quad$ $z = 0$ $\quad$ $z = \dfrac{87 - 85}{13.20}$

$z = -0.38$ $\quad$ $z = 0.15$

prob $(80 < x < 87)$

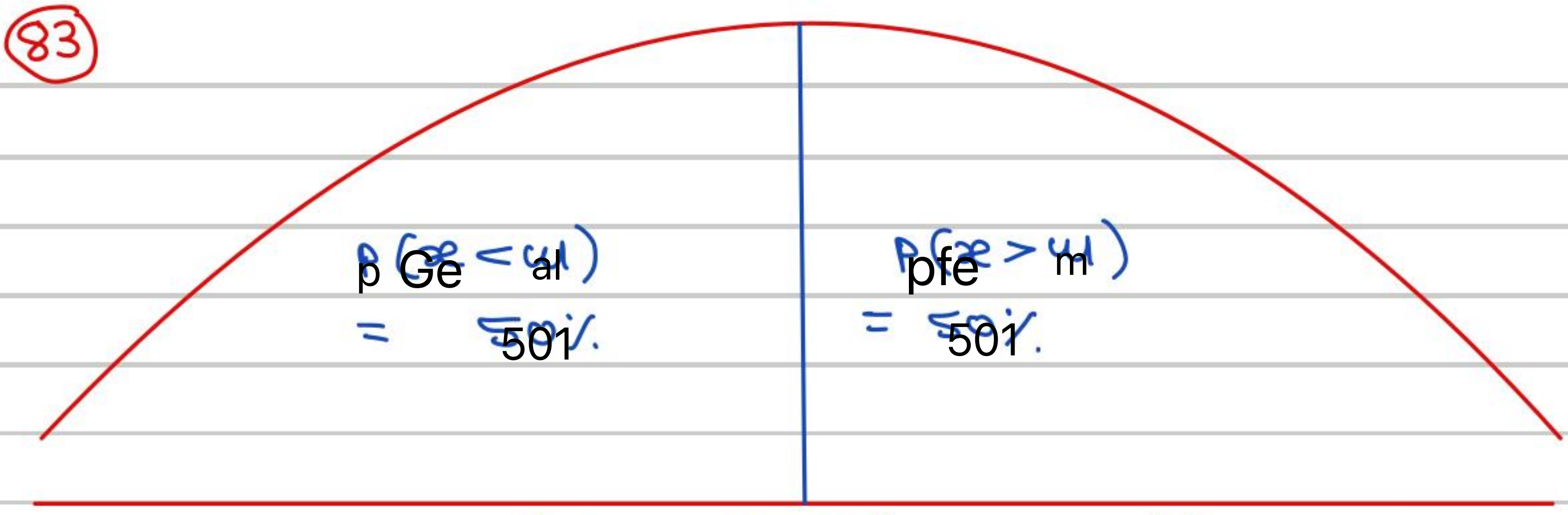
$= A(ABMN) + A(ABPQ)$

$= 0.14803 + 0.05962$

$= 0.20765$

(20.765%)

83



Z negative — $x < \mu$

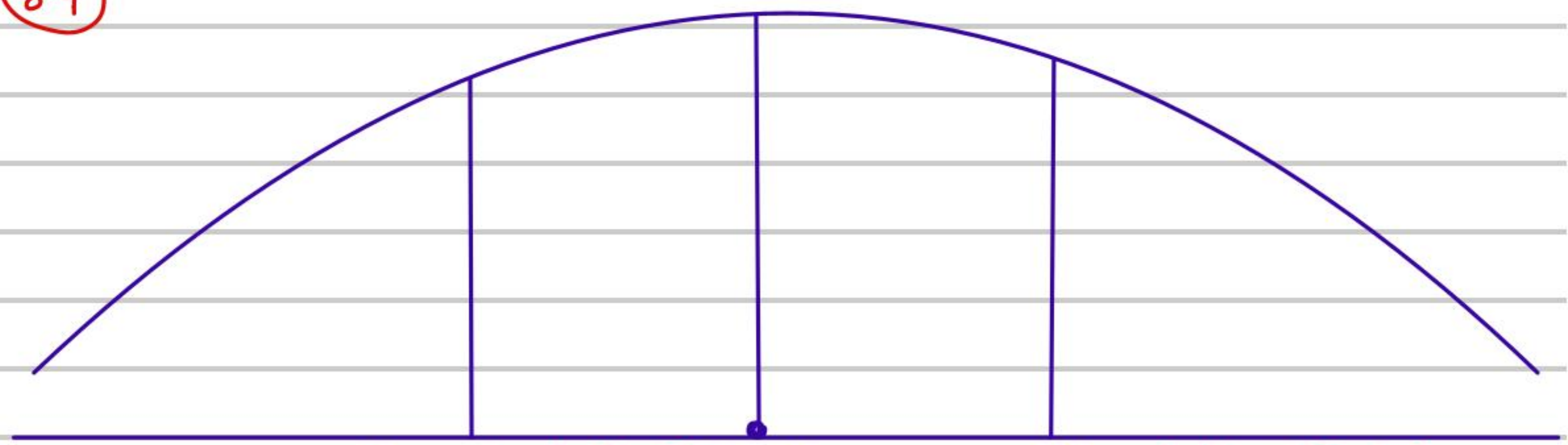
$x = AM$, $x = \mu$, $Z = 0$

Z positive — $x > \mu$

Z = Normal curve coefficient = $\left(\dfrac{x-\mu}{\sigma}\right)$

when	Z
$x = \mu$	0
$x > \mu$	positive
$x < \mu$	negative

84



$Q_1 = \mu - 0.6750\sigma$ $\qquad$ $Q_3 = \mu + 0.6750\sigma$

$x = AM = \text{Median} = \text{Mode}$
$= Q_2$

$$Q.D. = \left(\frac{Q_3 - Q_1}{2}\right) = \frac{\cancel{\mu} + 0.6750\sigma - \cancel{\mu} + 0.6750\sigma}{2} = \frac{1.350\sigma}{2}$$

$$Q.D. = 0.6750\sigma$$

i. Q.D. = 0.675 × S.D.

& M.D. = 0.80 × S.D.

QD is 67.50% of S.D. &
MD is 80% of SD

⟹ For normal Distribution.

(85) FOR NORMAL DISTRIBUTION

QD	MD	SD
26.875	31.85185	39.8148148148
67.50	80	100
63.1125	74.80	93.50
86.8888	102.9793185	128.724148148
75.3658	89.3224	111.653037037
1.29515625	1.535	1.91875
1.58203125	1.875	2.34375
6.075	7.20	9.00
8.690625	10.30	12.875
11.694375	13.86	17.325
53.98	63.976296	79.9703703703
0.6750m	0.80m	m
0.84375q	q	1.25q
j	1.185185j	1.48148148j

QD < MD < SD

OR

SD > MD > QD

points to be remembered

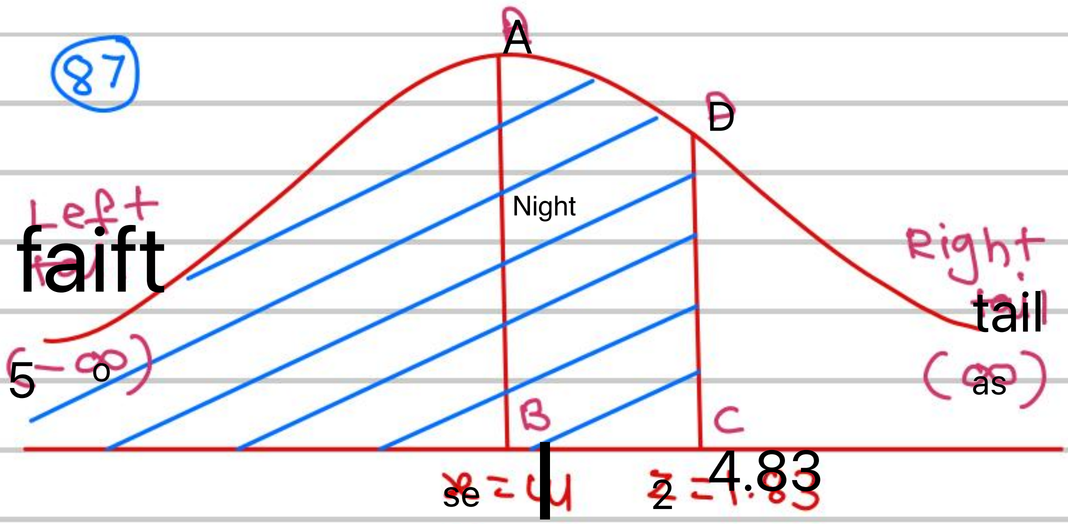
$QD = 0.6750\ SD$

$MD = 0.80\ SD$

86

Q_3	Q_1	QD	MD	SD
25	16	4.50	5.33333	6.666666
18	7.20	5.40	6.40	8.00
255	189	33	39.11111	48.88888
69.80	36.53	16.635	19.71555	24.64444
80	20	30	35.5555	44.44444
50.81	20.33	15.24	19.062222	22.57777
1.86	1.26	0.30	0.35555	0.444444
0.26	0.1205	0.06975	0.082666	0.1033333

87



shaded area

$= 0.50 + A(ABCD)$

$= 0.50 + 0.46638$

$= 0.96638$

$= 96.638\%$

$\phi(1.83) = 0.96638$

$\phi(1.83)$ represents area from $-\infty$ to 1.83

88 $\phi(a)$ represents area from $-\infty$ to a

$z = -0.67$ $\quad x = \mu$, $z = 0$ $\qquad\qquad x = \mu$, $z = 0$ $\quad z = 0.67$

$\phi(-0.67) = 0.25143 \qquad\qquad \phi(0.67) = 0.74857$

$\phi(k)$ represents area from $-\infty$ to k

$\phi(-j)$ represents area from $-\infty$ to $-j$

(89) $\phi(a)$ represents area from

(a) $-\infty$ to a $\qquad$ (b) a to ∞

(c) $-\infty$ to $-a$ $\qquad$ (d) $-\infty$ to a

(90) If $Q_3 = 80$, $Q_1 = 35$ Find AM, QD, SD, MD, Median, Mode for Normal Distribution.

$\Rightarrow$

① $Q.D. = \left(\dfrac{Q_3 - Q_1}{2}\right) = \dfrac{80-35}{2} = 22.50$

② $Q.D = 0.6750 \times SD \quad \therefore SD = \dfrac{22.50}{0.6750} = 33.333333$

③ $MD = 0.80 \times SD \quad \therefore MD = 0.80 \times 33.33333 = 26.66666$

(4) $\mu + 0.6750\sigma = 80$

$\mu - 0.6750\sigma = 35$

$2\mu = 115$ $\mu = \dfrac{80+35}{2}$

$\boxed{\mu = 57.50}$

$\therefore$ AM = Median = Mode = 57.50

$\therefore$ Median $= \dfrac{Q_3 + Q_1}{2}$

(91) For normal distribution If $Q_3 = 26.75$, $Q_1 = 13.25$. Find AM, Median, Mode, QD, MD, SD.

$\Longrightarrow$

(1) Median $= \left(\dfrac{Q_3 + Q_1}{2}\right) = \left(\dfrac{26.75 + 13.25}{2}\right) = 20$

$\therefore$ AM = Mode = Median = 20

(2) $Q_3 = \mu + 0.6750\sigma$

$26.75 = 20 + 0.6750\sigma$

$\sigma = 10 = SD$

(3) $MD = 0.80 \times SD = 0.80 \times 10 = 8$

(4) $QD = 0.675 \times SD = 0.675 \times 10 = 6.75$

(92) $Q_3 = 200$, $Q_1 = 100$ Find AM, Median, Mode, MD, SD, QD for normal distribution.

$\Longrightarrow$ ① Median $= \left(\dfrac{Q_3 + Q_1}{2}\right) = 150$

AM = Median = Mode = 150

② $QD = \left(\dfrac{Q_3 - Q_1}{2}\right) = 50$

③ $SD = \dfrac{QD}{0.6750} = 74.074074074$

④ $MD = 0.80 \times SD = 59.259259259$

$$\boxed{SD > MD > QD}$$

(93) For Normal distribution If MD = 61.75
Find QD

$\Longrightarrow$ $MD = 0.80 \times SD$

$\therefore SD = \dfrac{61.75}{0.80} = 77.1875$

$QD = 0.6750 \times SD$

$= 0.6750 \times 77.1875$

$= 52.1015625$

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$$\boxed{\begin{aligned} QD &= 0.6750 \times SD \\ MD &= 0.80 \times SD \end{aligned}}$$

Must remember

(1) $QD = 0.6750 \times \dfrac{MD}{0.80} = 0.84375 \times MD$

$$\boxed{\therefore\ QD = 0.84375 \times MD}$$

(2) $QD = 0.84375 \times MD$

$\therefore\ MD = \dfrac{1}{0.84375} \times QD$

$$\boxed{MD = 1.1852 \times QD}$$

(3) $QD = 0.6750 \times SD$

$SD = \dfrac{1}{0.6750} \times QD$

$$\boxed{\therefore\ SD = 1.48148 \times QD}$$

(4) $MD = 0.80 \times SD$

$$\boxed{\therefore\ SD = 1.25 \times MD}$$

95 If $\mu = 30$, $\sigma = 2.50$ Find Q_3, Q_1, QD, MD, Median, Mode

$\Longrightarrow$

(1) AM = Median = Mode = 30

(2) $Q_3 = \mu + 0.6750\sigma = 30 + (0.675 \times 2.50)$
$= 31.6875$

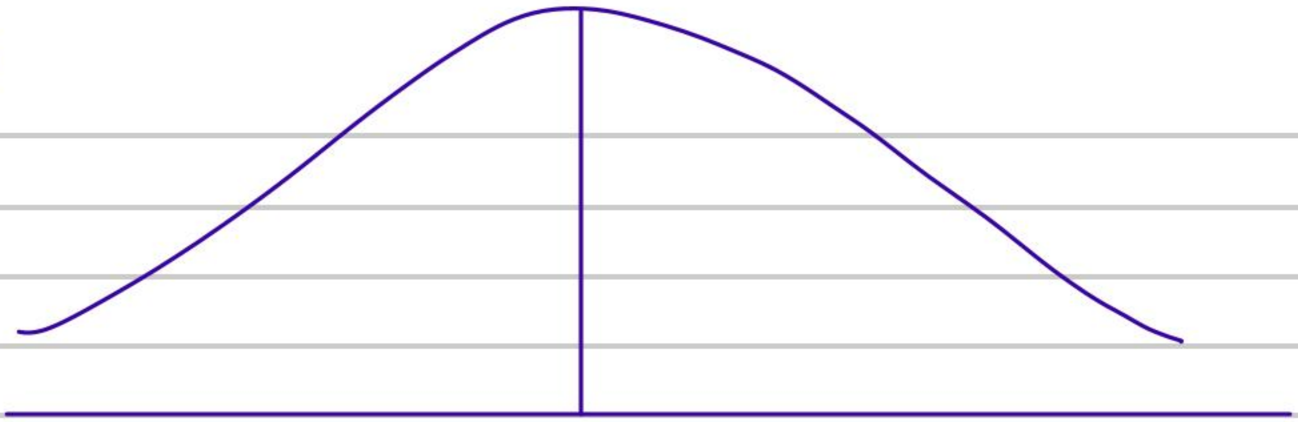
(3) $Q_1 = \mu - 0.6750\sigma = 30 - (0.675 \times 2.50)$
$= 28.3125$

(4) $Q.D. = \left(\dfrac{Q_3 - Q_1}{2}\right) = 1.6875$

OR $= 0.6750 \times 2.50 = 1.6875$

(5) $MD = 0.80 \times 2.50 = 2.00$

(96)



Points of inflexion of normal curve are $(\mu - \sigma)$ & $(\mu + \sigma)$

where curvature normal curve changes from concave to convexe and convexe to concave.

(97) If $\mu = 80$, $\sigma = 7.50$. Find points of inflexion of normal curve.

$\Rightarrow$

Points of inflexion are

$(80 - 7.50)$ & $(80 + 7.50)$

i.e. 72.50 & 87.50

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Summary – NORMAL DISTRIBUTION

1. It is a probability distribution applicable for continuous variables like Age, Height, Temp, wages, Income etc.
2. It is derived by Karl Gauss ∴ It is known as <u>Gaussian's Theorem</u>
3. It is based on assumption of Normality.
4. A continue variable is said to be normally distributed when 50% of obsns are less than AM & 50% obsns are more than AM. i.e. prob $(x \le \mu) = 50\%$. prob $(x > \mu) = 50\%$.
5. Normal curve is Bell shaped curve, symmetrical about AM.

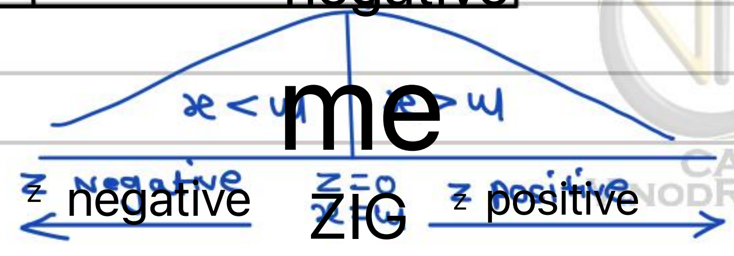
   Normal curve is symmetrical when $x = \mu$.

   when $x = \mu$ then skewness of curve is zero.

   (i.e curve is neither positively skewed nor Negatively skewed)
6. Left tail $= -\infty$, Right tail $= \infty$

   Two tails of normal curve will never touch Horizontal axis.
7. Z = Normal curve coefficient $= \left(\dfrac{x-\mu}{\sigma}\right)$
8.

when $x = \mu$	$z = 0$
when $x > \mu$	z = positive
when $x < \mu$	z = Negative



(9) AM = Median = Mode

(10) Median = $\left(\frac{Q_3 + Q_1}{2}\right) = \mu$ = Mode

(11) $Q_3 = \mu + 0.6750\sigma$

$Q_1 = \mu - 0.6750\sigma$

(12) Q.D. = $0.6750 \times SD$

M.D = $0.80 \times SD$

(13) $\phi(a)$ represents area from $-\infty$ to a

(14) Total area covered by normal curve = 1.00 = 100%.

(15) points of inflexion of Normal curve are $(\mu - \sigma), (\mu + \sigma)$

(16) μ, σ^2 are the parameters of normal distribution.

$\therefore$ It is a Bi-parametric distribution.

(17) Expected frequency = N × Expected probability

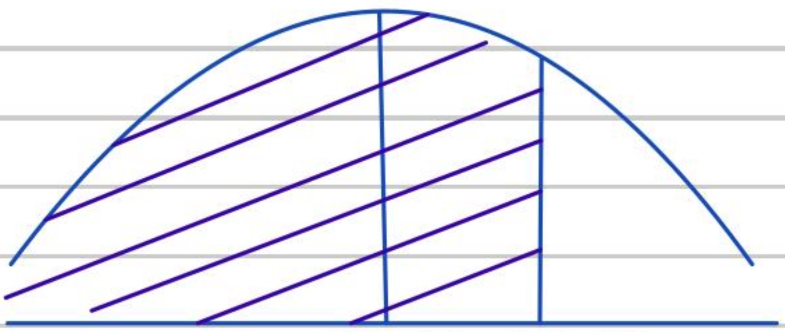
(18) probability that the continuous variable will assume a specific value is always zero.

(99) If $\mu = 500$, $\sigma = 80$

Find prob $(x < 540)$, prob $(x > 540)$, prob $(x \le 540)$

$\phi(0.50) = 0.69146$

$\Longrightarrow$



prob $(x < 540)$

$= 0.69146 = 69.146\%$

prob $(x > 540)$

$= 1 - 0.69146$

$= 0.30854 = 30.854\%$

$prob\,(x \le 590) = prob\,(x < 590) + prob\,(x = 590)$

$= 0.69146 + 0$

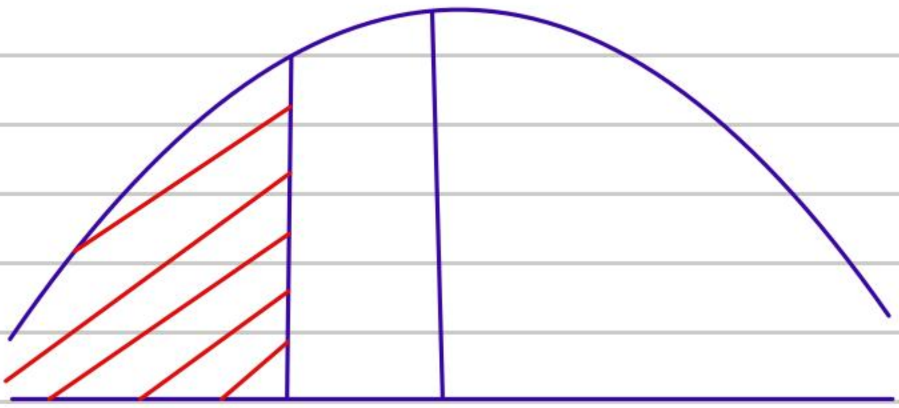
$= 0.69146$

$\therefore\ prob\,(x \le 590) = prob\,(x < 590)$

100) If $\mu = 10{,}000$, $\sigma = 1250$

Find $prob\,(x < 9250)$, $prob\,(x \le 9250)$

$\Longrightarrow$



$x = 9250$ $\qquad$ $x = 10{,}000$

$z = \dfrac{9250 - 10{,}000}{1250}$ $\qquad$ $z = 0$

$z = -0.60$

(1) $prob\,(x < 9250)$

$= 0.50 - 0.22575$

$= 0.27425 = 27.425\%$

(2) $prob\,(x \le 9250)$

$= 27.425\%$